Is the work of teaching geometry subject specific?

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Why? Of course, you could say, to the extent that one teaches geometry, that work is subject specific—but let's think beyond the obvious fact that the object of studies is a domain of mathematics and against the background that much pedagogical theory over the decades has spoken of the work of the teacher as capable of description by bracketing the subject matter (e.g., Hattie, 2009, p. 161-236). The notions of didactical contract and instructional situations provide a framework to argue that the work of teaching geometry is subject specific, beyond the obvious fact that the object of studies is a domain of mathematics.

Instructional situations, in particular, call for the geometry teacher to do some things to facilitate learning that other mathematics teachers or teachers of other subjects do not need to do. This observation can be tricky to make: Inasmuch as abstraction may allow us to see the same work being done in two very different manifestations, it would be possible to cast the work of teaching in such abstract terms that the differences across the teaching of different subjects might get elided. But the notion that the teaching of mathematics involves specific knowledge that aides teachers in doing their work in specific instructional situations, knowledge that is either possessed by individual teachers or recognized by teachers as being demanded by specific work, helps us discourage the use of such abstractions to describe the work of teaching (Herbst & Chazan, 2012; Herbst et al., 2010). We elaborate this paragraph in this section.

Consider two instructional situations—exploring a figure and calculating a measure. In both of them the teacher is called to create a representation of a figure for students to work with. In the case of exploring a figure, inasmuch as students will interact proximally with the representation and use those interactions to make assertions that instantiate target properties, we conjecture that in order to enable students' mathematical work the teacher would have to take great care in the creation of an accurate geometric representation, with preference to those that would not contain great risk of reading errors. This might mean drawing the representation with precise tools and thin strokes, as well as doing as much as possible to have measurements that are whole numbers or that involve common fractions (e.g., it is more likely that two congruent sides in a parallelogram will be 4.5 each rather than 4.55 each if students are expected to conjecture their congruence from measuring them). These actions on the part of the teacher might be interpreted as extreme attention to detail on the part of the individual teacher but they might also be read as what needs to be done to enable students to use their interactions with the figure to read target instances of the property at stake: If the diagram is accurate, the student would be able to consider the target property as a possibility; if the diagram is very accurate, the student would be able to confirm empirically their perception when they interact proximally, and would be encouraged to state the property even if still speaking of particulars (e.g., these sides are congruent). We contend that such attention to detail in creating a diagram for an exploration is an example of how the teaching of geometry is subject specific: The mathematical work students need to do with the diagram makes demands on what the teacher needs to do to set up such work.

In the case of calculating a measure, the teacher also needs to provide a representation, but this one is different and calls for different work on the part of the teacher. The teacher must create a diagram, but since the diagram will not be handled proximally, its drawing does not need to be very accurate. The drawing needs to be accurate enough for the student to perceive some properties read off the diagram as plausible, but such readings are not to be confirmed by measurement. So, for example if a teacher gives students a figure like Figure 1 it will be important that strokes $\overline{AD}$ and $\overline{BC}$ appear congruent though it is not needed that they are exactly the same length or that their length is a common number. It will be important, however, that the teacher also worries about other representations provided—the algebraic expressions that represent the said congruent sides need to be such that when equations are posed and solved, they will provide consistent measures. What is meant by *consistent measures* is heavily subject specific. It means in particular two things: One, that variables need to be consistently evaluated—that is, if *x* will take a value for one expression, it will take the same value in all expressions that use *x* in to represent dimensions in the same figure. It means also that findings about the measures of the figure obtained after equation solving, need to cohere with other geometric properties that are true about the figure they avowedly represent, e.g., a length cannot be negative or the legs of a right triangle cannot be longer than its hypotenuse.

AB =x+2
BC=5 - *x*
AC=2x+3
BD=3x+ 2

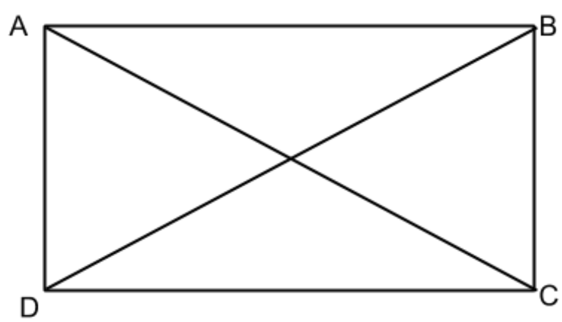

Figure 1. Calculate the lengths of the diagonals of the rectangle *ABCD*

Clearly, one could say that those cases of the work of teaching are just examples of the teacher creating the givens of a problem for students to be able to work on the problem, and even more abstractly, that those cases are just examples of the teacher creating the resources for an assignment so that students can do the assignment. But students could engage in the work even if the teacher did not create those representations in the manner described: A teacher could draw a diagram sloppily and get students to explore it; a teacher could attribute inconsistent algebraic expressions to the sides of a figure and get students to calculate the measures of its sides. That is, the actions the teacher does could still be described generically as creating the givens of a problem but be done in ways that breach the norm of the instructional situation they support. These breaches would impact the quality of the mathematical work students engage in (though not necessarily downgrade this quality). This observation suggests that the actions a teacher does need to be described in subject specific way, where the categories of subject specificity are identified by the norms of the instructional situation. To be clear, if we adopted a generic description and said that these are just cases of the teacher creating resources for a student assignment, we would need to accommodate within that description both (1) the case of a teacher that does so complying with the norms of the situation and (2) the case of a teacher who does so but breaching a norm (e.g., provides a diagram for an exploration but the diagram is inaccurate). To the extent that practitioners notice (or repair) breaches of norms like these (see Herbst, Aaron, Dimmel, & Erickson, 2013, for an example), we can say that at least for teachers, the grounds for the distinction we have made are not just different examples of the same abstract category, but actual information, in the sense of Bateson (1972), "a difference that makes a difference" (p. 315).

The prior paragraphs illustrate the elements of an argument for the claim that the work of teaching geometry is subject specific. Obviously, the criteria used to detect differences, whether these are summative measures of achievement and success or analyses of the qualities of the mathematical work, matters in deciding whether these are differences that make a difference. And obviously also, some of the subject specific differences that the notion of instructional situation permits us to detect are nested in general approaches to teaching (e.g., problem based instruction, direct instruction) that contribute by themselves to making or not making a difference (see Hattie, 2009). But having said that, to the extent that the work of teaching involves transacting students' work on tasks for claims on what they know or not know, some broad tasks of teaching emerge (e.g., creating work assignments, interpreting the students' work) that are intrinsically connected to the subject specific work that students do. The way a teacher carries out these tasks of teaching could be idiosyncratic (e.g., a teacher might always be sloppy in the assignments he or she provides), but we would not expect such idiosyncratic behavior to apply to the majority of professionals. Instead we'd expect that participants' recognition of the norms of the instructional situation that frames the work and their knowledge of the mathematics needed to enact such instructional situations would account for the variance in the ways teachers enact these tasks of teaching. As can be noted, teachers' knowledge of the mathematics they teach, geometry in this case, can be crucial to explain what teachers do.

References

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